

Calculators are not allowed

Answer the following questions (all items are weighted equally at 5 points each):

1. Use differentials to find an approximate value of $\frac{2.99}{\sqrt[3]{1 - (2.99)^2}}$

2. Let y be a function defined implicitly by the equation

$$x^2y + \tan \sqrt{x^2 + y} = 1,$$

find an equation of the tangent line to the graph of y at the point $(\frac{\pi}{4}, 0)$

3. Suppose f is a function defined on $(-\infty, \infty)$, $f'(x) = 1$ for all x and $f(2) = 3$. Use the Mean Value Theorem to show that $f(x) = x + 1$ for all x .

4. A piece of ice of the shape of a sphere is melting and its surface area, S , is given by, the time-dependent relation

$$S = 4\pi(25 - t^2)$$

If the sphere does not lose shape, then find the rate of change of its volume when $t = 4$ sec.

5. Let

$$f(x) = 3 - \frac{4}{x} - \frac{4}{x^2}.$$

(a) Find the intervals on which f is increasing or decreasing, and find the local extrema of f (if any).

(b) Find the intervals on which the graph of f is concave upward or concave downward, and find the points of inflection (if any).

(c) Find the vertical and horizontal asymptotes for the graph of f (if any).

(d) Sketch the graph of f .

(Good Luck!)